**BOND Valuation, Duration and Convexity**

Definition: A bond is a long-term promissory certificate showing a borrower owes a specified sum.

1. Indenture: is the contract between an issuer and the bondholder, which set forth all the obligations of the issuer.

2. Term to Maturity: number of years over which the issuer has promised to meet the conditions of the obligations.

1. Principal and Coupon Payment.

**Bond Valuation:**

* + a. Par =face or terminal value
	+ b. C =Coupon payment, C = Par x CR
	+ c. CR =Coupon Rate
	+ d. T =Maturity Date
	+ e. R=Discount rate, required rate of return, Interest rate.
	+ f. Pb =Price of a bond

**Forms of Bonds:**

**1. Pure Discount Bond** 

**2. Level - Coupon Bonds**



**3. Consol. (Perpetuities):**

**CONVENTIONAL YIELD MEASURES**

1. Current Yield

2. Yield to Maturity

3. Yield to Call

**Yield to Maturity**

The valuation model set forth in equation 3 can also be used to find a bond’s *yield to maturity.*

The bond’s yield to maturity is the bondholder’s expected rate of return if the bond is held to maturity.

Finding the expected rate of return (yield to maturity) is a trial-and-error process. Below is an approximate method of calculating the yield to maturity:

 

**Yield for a Portfolio**

 The yield for a portfolio of bonds is not simply the average or weighted average of the yield to maturity of the individual bond issues in the portfolio. It is computed by determining the cash flows for the portfolio and determining the interest rate will make the present value of the cash flows equal to the market value of the portfolio.

RISKS ASSOCIATED WITH INVESTING IN BONDS:

Bonds may expose an investor to one or more of the following risks:

1. Interest-Rate Risk

2. Reinvestment Risk

3. Call Risk

4. Default Risk

5. Inflation Risk

6. Exchange Risk

7. Liquidity Risk

**Price-yield relationship and Bond Pricing Principles**

A bond's risk stems mainly from possible changes in interest rates.

An increase in interest rates reduces the price of a bond, **price effect**, but increases the reinvestment rate for coupon payment, **reinvestment effect.**

**Reinvestment Rate Assumption**



**Principle 1. Bond prices move inversely with interest rates.**

**Principle 2. The long the maturity of a bond, the more sensitive is its price to a change in interest rates, holding other factors constant. Example 1**

|  |
| --- |
| Bond A: 30-years, 10% coupon rate |
| Bond B: 5-years, 10% coupon rate |

|  |  |  |
| --- | --- | --- |
|  Interest Rates  | % Price Change |   |
|   | 8% | 10% | 12% | Yield Falls | Yield Rises  |
| Bond A | 1226 | 1000 | 838.4 | 22.62% | -16.16% |
| Bond B | 1081 | 1000 | 926.4 | 8.10% | -7.36% |

**Principle 3. The price sensitivity of bonds increases with maturity, but at a decreasing rate**. **Example 2**

Bond A: 30-years, 10% coupon rate Bond C: 20-years, 10% coupon rate

Bond D: 10-years, 10% coupon rate

|  |  |  |
| --- | --- | --- |
|  Interest Rates  | % Price Change | Difference In Price Change |
|   | 10% | 8% |
| Bond A | 1000 | 1226.2 | 22.62% |  2.83% |
| Bond C | 1000 | 1197.9 | 19.79% |
| Bond D | 1000 | 1135.9 | 13.59% | 6.20% |

**Principle 4. The lower the coupon rate on a bond, the more sensitive is its price to a change in interest rates, holding other factors constant. Example 3:**

Bond E: 30-years, 14% coupon rate Bond F: 30-years, 6% coupon rate

|  |  |  |
| --- | --- | --- |
|  |  Interest Rates  | % Price Change |
|   | 10% | 8% |   |
| Bond E | 1379 | 1678.7 | 21.77% |
| Bond F | 621.4 | 773.8 | 24.53% |

**Principle 5. For a given bond, the capital gain caused by a yield decrease exceeds the capital loss caused by a yield increase of the same magnitude. Example 4:**

Bond A: 30-years, 10% coupon rate

|  |  |  |  |
| --- | --- | --- | --- |
|   |   | Price When Rate  | % Price |
| Interest | Price | Falls | Rises | Increase | Decrease |
| Rate |   | 1% | 1% |   |   |
| 8% | 1226.2 | 1374.2 | 1103 | 12.07% | 10.03% |
| 10% | 1000 | 1103.2 | 912.8 | 10.32 | 8.72 |
| 12% | 838.4 | 912.8 | 774.5 | 8.87 | 7.62 |

**The Need for a Summary Measure-Duration**

The five principles of bond pricing are all very important for understanding bond investing. However, since each principle assumes that all other factors are being held constant, except the one under examination, it is still difficult to compare the price sensitivity of different bonds.

**Duration** is a gauge of a bond's basic price volatility. It is a measurement of how long, in years, it takes for the price of a bond to be repaid by its internal cash flows.

**Macaulay duration** is the weighted term to maturity expressed in years where the weights are the present value of the cash flows occurring in those years.

**Macaulay Duration** 

Or 

**Modified duration** is the Macaulay duration divided by (1 + semi-annual yield) for bonds with semi-annual payments. Equivalently the first derivative of the bond price-yield equation with respect to yield divided by the price (present value) of the bond produces the same modified duration. Modified duration measures bond price volatility and provides an estimate of the rate of change in bond price due to a change in yield. As such, modified duration measures a bond's sensitivity to interest rate changes and its exposure to interest rate risk.

**Modified Duration** 

**The Duration Price Change Formula**

There is another equation that expresses duration as the negative of elasticity of the bond's price with respect to a change in the discount factor.



By rearrange the above equation:



**Reinvestment Rate and Duration**

Assume a four-year bond that pays $100 a year, has YTM of 10%, and a Par=$1000. We know the bond is currently trading at $1000. If the interest jumps to 12%, then the price is 939.25.

C=$100 Par=$1000 YTM=0.12 T=4 Price=$(939.25)

If your holding period is only a day, you will lose 6.075% of the value. But, if you hold it for 4 years, then you are going to be better off, 10.26% (see the table below). Thus, for one day holding you are worse off, for a four-year holding period you are better off. Therefore, somewhere between a holding period of one day and holding period of the maturity of the bond you would anticipate a holding period where the **price effect** and the **reinvestment effect** just offset one another. Duration is the holding period that balances the price effect against the reinvestment effect.

 Future Reinvestment Rate and Estimation of Duration

Period Coupon (1+12%)t (1+8%)t t. C 1/(1+10%)t 1/(1+10%)t (t. C)

1 100 140.49 125.97 100 0.9091 90.91

2 100 125.44 116.64 200 0.826 165.29

3 100 112.00 108.00 300 0.751 225.39

4 100 100.00 100.00 400 0.683 273.20

4 Par 1000 1000 4000 0.683 2732

**Total 1477.93 1450.61** Total 3486.8

Price 1000.00 1000.00 **Duration =(3486.8/1000)=3.49**

**Realized YTM 10.26% 9.75%**

**YTM=10% YTM=6% YTM=12%**

**Price** $1,000.00 $1,138.60 $939.25

**Percent change** **13.86% 6.1%**

Using the duration, if the yield on the bonds falls to 6%, according to the duration formula, the price of the bond is expected to rise by 12.69%, or to a price of $1126.90.

The actual new price of the bond will be $1138.6 (13.86%) which is higher than predicated. This represents a substantial error, 8.44% (1-12.69%/13.86%). A bond’s convexity, which is a measure of the degree of curvature in its price-yield relationship, can be used to try to correct for these types of predication errors.

**The Reinvestment Rate Effect**

The reinvestment rate (RR) over time has a significant effect on both the ending value and the realized yield to maturity (RYTM) for a bond as we have shown above. The effect of the reinvestment rate, the holding period (H) of a bond and the duration (D) on the realized yield to maturity RYTM can be shown as:



When the reinvestment rate is 12%, RYTM is 10.26%.

**Duration and Bond Price Volatility**

* Bond price movements will vary proportionally with modified duration for small changes in yields
* An estimate of the percentage change in bond prices equals the change in yield time modified duration

Price change based on modified duration 

For small changes this will give a good estimate, but this is a linear estimate on the tangent line

**Characteristics of Duration**

* Duration of a bond with coupons is always less than its term to maturity because duration gives weight to these interim payments
* A zero-coupon bond’s duration equals its maturity
* An inverse relation between duration and coupon
* A positive relation between term to maturity and duration, but duration increases at a decreasing rate with maturity
* An inverse relation between YTM and duration
* Sinking funds and call provisions can have a dramatic effect on a bond’s duration

**Trading Strategies Using Duration**

* Longest-duration security provides the maximum price variation
* If you expect a decline in interest rates, increase the average duration of your bond portfolio to experience maximum price volatility
* If you expect an increase in interest rates, reduce the average duration to minimize your price decline
* Note that the duration of your portfolio is the market-value-weighted average of the duration of the individual bonds in the portfolio

**Duration and Convexity**

**Convexity:** Duration measures the percentage change in a bond’s price given a change in the bond’s yield. Duration also measures the slope of a line tangent to the convex price-yield relationship shown in the figure below.

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 ***Convexity*** **measures the rate of change of a bond’s duration as the level of the bond’s yield changes.** Convexity can be estimated by:





**Determinants of Convexity**

* Inverse relationship between coupon and convexity
* Direct relationship between maturity and convexity
* Inverse relationship between yield and convexity

**Properties of Convexity**
If two bonds offer the same duration and yield but one exhibits greater convexity, changes in interest rates will affect each bond differently. A bond with greater convexity is less affected by interest rates than a bond with less convexity. Also, bonds with greater convexity will have a higher price than bonds with a lower convexity, regardless of whether interest rates rise or fall. This relationship is illustrated in the following diagram:



**Price-Yield Relationship for Bonds**

* The graph of prices relative to yields is not a straight line, but a curvilinear relationship
* This can be applied to a single bond, a portfolio of bonds, or any stream of future cash flows
* The convex price-yield relationship will differ among bonds or other cash flow streams depending on the coupon and maturity
* The convexity of the price-yield relationship declines slower as the yield increases
* Modified duration is the percentage change in price for a nominal change in yield

**Modified Duration-Convexity Effects**

* Changes in a bond’s price resulting from a change in yield are due to:
	+ Bond’s modified duration
	+ Bond’s convexity
* Relative effect of these two factors depends on the characteristics of the bond (its convexity) and the size of the yield change
* Convexity is desirable

A bond’s convexity can be used to as a supplement to the duration when predicating how a bond’s price will change with a given change in yield. That is:

